Transmissivity of Glass

Introduction

In this project we investigate the radiative properties of two particular types of glass: one of them a standard glass and the other what is called a "low – E" (for emissivity) glass. Stop by any glass shop and you can pick up literature on various brands of the latter. You will find that many modern windows are made of two panes of glass, at least one of them having a low-E coating, and with, perhaps, an inert gas like argon or krypton sealed in the gap between them. In such glass manufactured for residential applications, the special coating is usually not visible to the eye. They are often visible on the windows of office buildings. Some coatings are metallic and may interfere with cell phone reception within a building. Polyester films don't interfere with your cell phone. A data file of the spectral transmissivity (τ_λ) is supplied for two representative glasses, and we will determine the total transmissivity of each glass for both solar (short) and terrestrial (long) wavelength radiation.

Recall that unlike total emissivity, which is a function of the temperature of the radiating surface itself, the total transmissivity (as well as total absorptivity and reflectivity) are functions of the irradiation. That is, the integration of the spectral value to get the total (or average) value must be weighted using the spectrum corresponding to the source of the radiation. Here we will take the sources to be blackbodies having two particular temperatures, specifically that of the sun and another corresponding to everyday terrestrial temperatures. We will do the integration roughly using the conventional, tabulated blackbody radiation functions and then follow up with a more accurate numerical integration using Simpson's 1/3 rule.

For both glasses you will compute quite different values of total transmissivity depending on whether the incoming radiation is high temperature solar (mostly short wavelength) or low temperature terrestrial (long wavelength) in origin. This significant difference in transmissivity to short vs. long wavelength radiation leads to what is known as the "greenhouse effect," a term which more recently has been extended to the analogous effect in the atmosphere caused by the accumulation of CO_2, water vapor and other so-called "greenhouse" gases.

Learning Objectives

By the time you have completed this assignment, you will be able to:

1. Use terms like short wavelength radiation, visible radiation, long wavelength radiation, heat, low-emissivity glass, etc., intelligently,

2. Explain how a total property is determined from a spectral property and why weighting by the appropriate function is needed in the integration,

3. Use the tabulated blackbody radiation functions,

4. Apply Simpson's rule to the integration of any particular function,

5. Explain how low-E glass may be used to achieve desired energy conservation goals.

The Problem

Data for the transmissivity of two glasses, one standard (uncoated) and the other low-emissivity, as a function of wavelength is given in the second sheet of the Planck's Law workbook. That data is plotted below. Both regular and low-E glass have high transmissivity (τ = 0.8 - 0.9) in the visible portion of the spectrum ($\lambda \approx$ 0.4 - 0.7μm). The transmissivity of the low-E glass is seen to drop off much faster than that of ordinary glass in the near infrared region. That means that with low-E glass the inside of the house still receives the visible sun light but with less overall solar heat gain. It is not obvious where the term "low-E" comes from, but since $\alpha + \rho + \tau = 1$, low transmissivity and high reflectivity imply low absorptivity, which means low emissivity, since $\alpha_\lambda = \varepsilon_\lambda$. "Low-E" thus refers strictly to the behavior of the glass in the infrared (long wavelength) part of the spectrum. If one were doing a radiative heat transfer analysis for the interior of a room (as discussed in the next chapter), the window would be considered an opaque surface with surface resistance $= \dfrac{\rho_i}{A_i \varepsilon_i}$. Then high reflectivity and low emissivity implies a high resistance to radiative heat transfer. High surface radiative resistance means less heat loss from the interior of the house during the heating season.

In your calculations assume the sun is a blackbody source at 5800K and produces an irradiation of 1100 W/m^2 on the glass - which might be the windshield of your car. (This value of the irradiation (G) includes the effect of attenuation in the earth's atmosphere, which reduces it from the value of the solar constant (1353 W/m^2). The solar constant corresponds to that which arrives per square meter normal to the sun's rays at the orbital radius of the earth from a blackbody source having the diameter of the sun and at the sun's apparent temperature. The terrestrial source is assumed to be black and at 300K.
You are to do the following and comment on your findings:

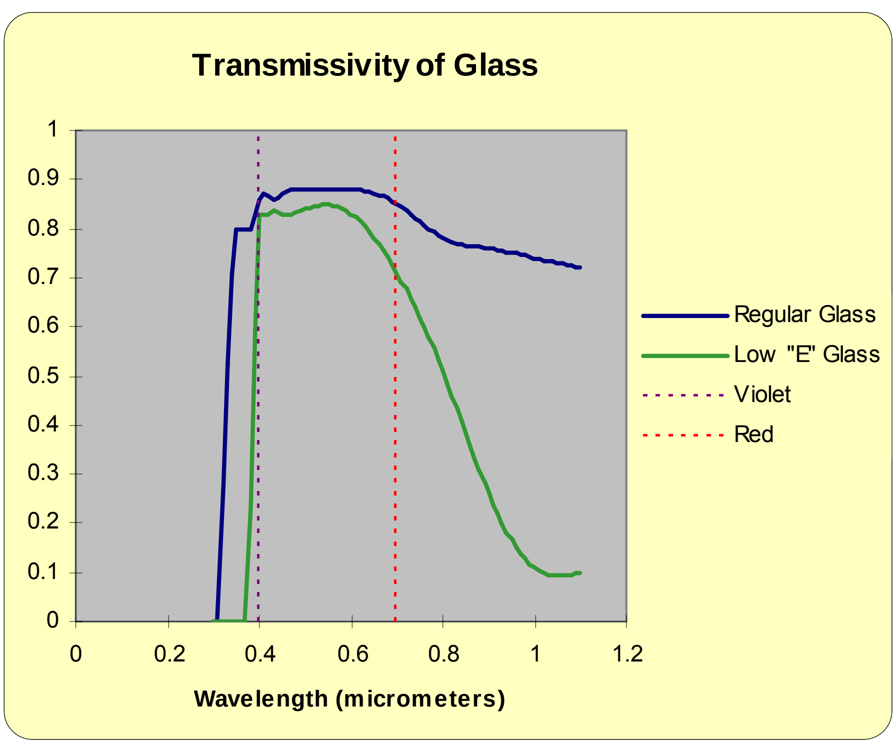

Figure1. Spectral transmissivity of regular and "Low E" glass

1. Access the Planck's Law workbook. The second worksheet includes a table of the transmissivity data for both glasses as seen in graphical form above.

2. Approximate both transmissivities as constant over a particular wavelength interval (make a good "eyeball" estimate based on the plots) and 0.0 outside that range, i.e., as a "boxcar" function. Sketch your function for both types of glass in your notes.

3. Using either the blackbody radiation function table in the text or the integration function of the Planck's law workbook with the boxcar functions you have chosen, estimate the total transmissivity of both glasses to irradiation from the sun. Assume the sun is a blackbody at 5800K. (See Equation 1 in the Appendix.) With the boxcar function approximation you can break the spectrum $0 \leq \lambda \leq \infty$ into three segments: $0 \leq \lambda \leq \lambda_1$ where τ and thus the integral are zero; a second interval in which τ is a constant and can be brought outside the integral and a third section $\lambda_2 \leq \lambda \leq \infty$ where τ and thus the integral are again zero.

4. Do this calculation again to estimate the total transmissivity of both glasses to the irradiation from a terrestrial source. Assume a blackbody at 300K. A table for recording your results is given below.

5. Now use Simpson's 1/3 Rule as described in the Appendix to do a numerical integration. Since you will be using the exact values from the tabulated data rather than the boxcar function you used earlier, your Simpson rule results should give better estimates. Now you should now have eight total transmissivities:

	Regular Glass	Regular Glass	"Low E" Glass	"Low E" Glass
	B.B.Rad. Func.	Simpson's Rule	B.B.Rad. Func.	Simpson's Rule
Solar Radiation (5800K)				
Terrestrial Radiation (300K)				

Computed Transmissivities

6 Using the transmissivity values found in Step 5 (which are presumably more accurate than those found in Step 3), find the total solar energy (W/m^2) transmitted through both glasses. Use the value of G = 1100 W/m^2 given above for solar irradiation.

7. Again using your values from Step 5, find the total energy transmitted from the terrestrial source (blackbody at 300K) for both glasses (W/m^2). Assume the source and the glass are two closely spaced parallel plates.

	Regular Glass	"Low E"
Solar Radiation		
Terrestrial Radiation		

Transmitted Radiation (W/m^2)

8. Speculate on how you might profitably incorporate low-E glass into the design of a home. Consider both summer and winter scenarios and northern and southern climates. Include sketches.

References

1. *2001 ASHRAE Handbook - Fundamentals*, American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc. Atlanta (2001).

Note: Some sources, including the ASHRAE Handbook, make a distinction between the "ivity" ending used here, and the "ance" ending. Technically the transmissivity, reflectivity, etc. refer to inherent properties of a bulk sample of the material, while the "ance," e.g., transmittance, refers to the property of a specific sample or thickness of a substance or combination of substances.

2. Çengel, Y.A., *Heat and Mass Transfer – A Practical Approach*, 3rd Ed., McGraw-Hill, 2007, pp. 679-688.

3. Chapra, S.C., and Canale, R.P., *Numerical Methods for Engineers*, 2nd Ed., McGraw-Hill, New York (1988), pp 490-494.

Appendix - The Numerical Integration

Problems like that given here are discussed in all heat transfer textbooks, but in those problems the variation of the transmissivity with wavelength is taken to be a very simple boxcar function - e.g., a constant over a particular wavelength range and identically 0.0 or another constant over the rest of the spectrum. Evaluating total properties can then be done easily using the blackbody radiation function table in the text or the integrate function in the Planck's Law workbook. In order to force you to do a numerical integral of the following definition of the total transmissivity:

$$\tau = \frac{\int_0^\infty \tau_\lambda(\lambda)G_\lambda(\lambda)d\lambda}{\int_0^\infty G_\lambda(\lambda)d\lambda} \tag{1}$$

we have given you more detailed transmissivity data at a large number of equally spaced points. Assume zero transmissivity outside the range of the given data. Since the irradiation is assumed to come from a blackbody source, then the weighting function,

$$G_\lambda(\lambda) = E_{\lambda,B}(\lambda,T) = \frac{C_1}{\lambda^5\left[e^{\frac{C_2}{\lambda T}}-1\right]} \tag{2}$$

Here C_1 = 3.742x10^8 $W\cdot\mu m^4/m^2$ and C_2 = 1.439x10^4 $\mu m\cdot K$. The Planck's Law workbook has a VBA function already prepared for this expression and you can use it in your calculations exactly as if it were a *supplied* function like the sine or cosine.

You can use Simpson's 1/3 rule for the integration:

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \ldots + 2y_{n-2} + 4y_{n-1} + y_n\right] \qquad (3)$$

Here y_0, y_1 … y_n are the ordinates of the curve y = f (x). (Here in the numerator of Equation 1 f(x) is the product $\tau_\lambda(\lambda)E_{\lambda,B}(\lambda,T)$) at the x values x_0 =a, x_1 = a + h, ... x_n = a + nh = b. You may recall that the *Trapezoidal Rule* for numerical integration assumes a linear function between each pair of points. Simpson's 1/3 rule assumes a quadratic relation for each trio of adjacent points; that is where the sequence 1,4,2,4,… comes from. Alternatively Simpson's 1/3 rule may be written:

$$\int_a^b f(x)dx \approx (b\text{-}a)\frac{f(x_o) + 4\sum_{i=1,3,5}^{n-1} f(x_i) + 2\sum_{j=2,4,6}^{n-2} f(x_j) + f(x_n)}{3n} \qquad (4)$$

For the given data file n = 80, i.e., there are 81 data points. (You need an even number of intervals in order to use Simpson's 1/3 Rule.) Note that the integral in the denominator of Equation 1 above should equal σT^4. This suggests a way to check your implementation of Simpson's Rule (realizing that you do have to cut it off somewhere short of infinity) using the table of blackbody radiation functions given in the text.